

The asynchronous machine model

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1 INTRODUCTION

The asynchronous machine has been modelled with a combination of impedances and power injection, so that its behaviour in load flow and short-circuit calculations produces a sufficient precise result. The impedances and injections are calculated from the machine parameters. This document describes the modelling and the implementation in Vision *Network Analysis*.

2 MODEL

The asynchronous machine stator creates a rotating electromagnetic field, when connected to a three phase AC network. The rotor windings are not connected to an external network. The electromagnetic torque can only exist if a rotor current flows, which will flow if the rotor revolves with an other speed than the electromagnetic field.

The stator rotating electromagnetic field revolves with the angular speed: ω/p . If the rotor revolves with another speed, an EMF will be induced in the rotor windings. If the rotor windings are short-circuited, a current will flow, creating an electromagnetic torque. Since the torque producing rotor current is produced by induction, the asynchronous machine also is called "induction machine". The difference in speed, relative to the synchronous speed, has been defined as slip:

$$s = \frac{n_0 - n}{n_0} \tag{1}$$

The rotor can be made with or without terminals to the rotor windings. The rotor with terminals has the possibility to connect a resistor in the rotor circuit or to connect a wye-delta switch to reduce motor starting currents. The rotor without terminals can be produced with windings, consisting of rods, connected to short-circuiting rings at both sides of the rotor: a "squirrel cage".

At normal load the rotating electromagnetic field between rotor and stator approaches the field in no-load conditions. It can be stated that in normal conditions the flux rotor and stator is constant. Only in case of heavy overload the electromagnetic field will reduce. At increasing stator current the stray field increases, decreasing the electromagnetic field. That is why the stator resistance R_l and the stray reactance $X_{l\sigma}$ may not be neglected in motor starting conditions. This phenomenon is analogue to the transformer, where the flux remains constant in normal conditions, but decreases in situations of heave overload. So the asynchronous machine can be compared to the transformer.

The asynchronous machine power balance is as follows:

$$P_1 = P_{mech} + P_{Cu2} + P_{Cu1} + P_{ext1} + P_{Fe} \tag{2}$$

With:

$$P_1 = 3 \cdot U_{phase} \cdot I_{phase}^* \cdot \cos(\varphi) \tag{3}$$

And:

- P_{mech} : mechanical power
- P_{Cu2} : copper loss in the rotor circuit
- P_{Cu1} : copper loss in the stator circuit
- P_{ext1} : extra loss in the stator circuit
- P_{Fe} : iron loss

The asynchronous machine electrically behaves like a transformer, with a secondary load equal to $R_2(I-s)/s$. A changing speed, from stand still ($s=1$) to synchronous speed ($s=0$), is electrically equal to changing the transformer load from short-circuit to no-load. In that case the load resistance changes from zero to infinite.

Like the transformer, the asynchronous machine has a T-model schematic. Analogously the model has a stator main field reactance $X_{1h} = \omega L_{1h}$. In this model the rotor circuit has been transformed to the stator circuit. Next diagram shows the model.

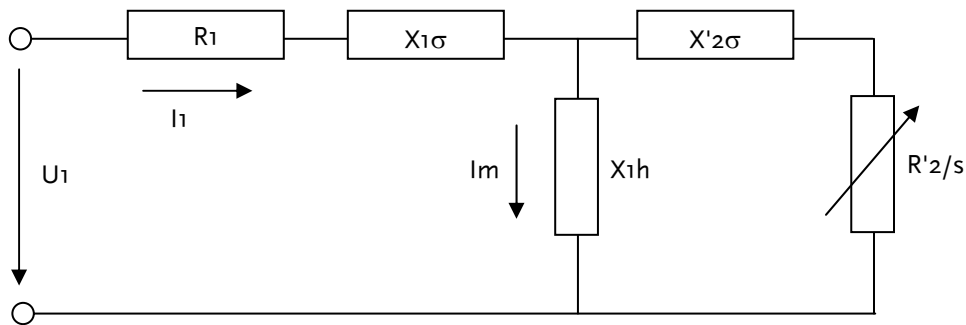


Figure 1 Model of an asynchronous machine

In this diagram:

- R_1 : stator resistance
- $X_{1\sigma} = \omega L_{1\sigma}$: stator stray field reactance
- $X_{1h} = \omega L_{1h}$: stator main field reactance
- $X'_{2\sigma}$: rotor stray field reactance, transformed to the stator
- $R'_{2/s}$: load, transformed to the stator

In Vision Network Analysis, the asynchronous machine has been modeled according to this model, neglecting the stator resistance and the stator stray field reactance. The model is applicable for normal load conditions and not for small machines.

Using the voltage and current equations, for each load condition the vector-diagram can be constructed, indicating the phase voltage and the phase current. The current diagram is the projection of the vector I_1 (the stator current) for all machine load conditions. The current diagram is complex.

Assumptions are:

- U_1 constant
- R_1 is zero
- R_2 and all reactances are constant
- the iron loss is zero

Based on these assumptions the current diagram is a circle and has been called the "Heyland Circle". From this circle it is possible to determine the currents, power, losses, electromagnetic torque and slip for all load conditions. The accuracy depends on the correctness of the assumptions made.

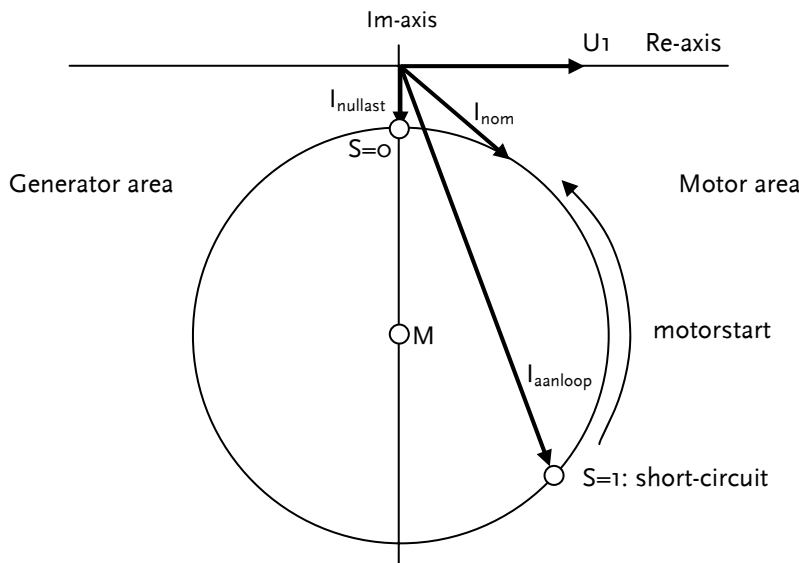


Figure 2 Heyland circle for an asynchronous machine

The circle centre is M. Clearly visible are the no-load point ($s=0$) and the short-circuit point ($s=1$) from where the motor starts. The normal load working area is the relative small part of the circle between $s=0$ and $s=s_{nom}$, at rated current I_{nom} . The area at the right side of the imaginary axis is the motor area of the machine. The left side area is the generator area, with negative slip.

The normal working area between no-load and rated load reflects a small part of the circle only. Beyond rated load the assumptions become less valid. In *Vision Network Analysis* the model is used from no-load to 125% of rated load, which is sufficient for most cases. The motor starting behaviour has not been modeled with the diagram, but with the measured I_s/I_{rated} ratio and the motor starting power factor. Also the motor efficiency and the power factor are measured for several load conditions. Using these measurements the Heyland circle can be determined, and finally the machine parameters for load flow studies.

3 IMPLEMENTATION

3.1 The model

Many computer programs model the asynchronous machine as a constant power load. The model in *Vision Network Analysis* accurately models the dependence of mechanical load and terminal voltage. The model neglects the stator resistance. Next diagram illustrates the model.

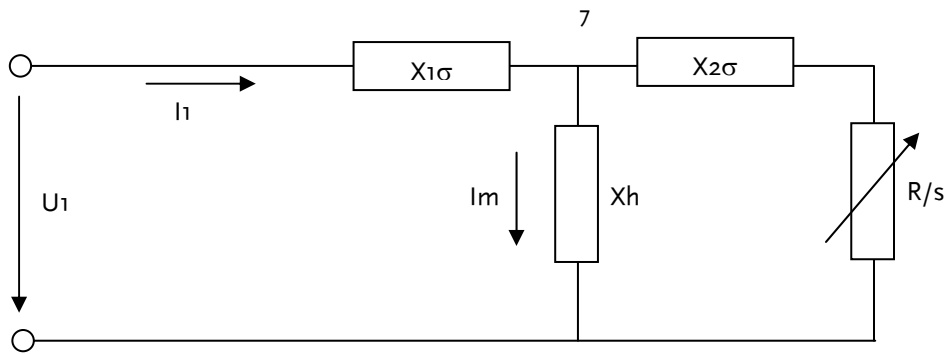


Figure 3 Model of the asynchronous machine in Vision Network Analysis

The model parameters can be used to construct the Heyland circle. It is the projection of the terminal admittance:

$$Y = \frac{1}{jX_{1\sigma} + \frac{1}{\frac{1}{jX_h} + \frac{1}{\frac{R}{s} + jX_{2\sigma}}}} \tag{4}$$

For a varying slip from nearly zero (synchronous speed; s approaches zero) the equation yields:

$$Y_{s \rightarrow 0} = \frac{1}{j(X_1 + X_h)} \tag{4a}$$

For a slip approaching to infinite the equation yields:

$$Y_{s \rightarrow \infty} = \frac{1}{j(X_{1\sigma} + X_h // X_{2\sigma})} \tag{4b}$$

Now the Heyland circle can be constructed. The circle crosses the imaginary axis on the points $-1/(X_{1\sigma} + X_h)$ and $-1/(X_{1\sigma} + X_h // X_{2\sigma})$. The circle centre also is on the imaginary axis, right in the middle of the two points. The circle radius $(1/2X)$ equals:

$$Radius = \frac{1}{2X} = \frac{1}{2} \left(\frac{1}{X_{1\sigma} + X_h // X_{2\sigma}} - \frac{1}{X_{1\sigma} + X_h} \right) \tag{4c}$$

Now that the Heyland circle has been drawn, the parameters to calculate the actual reactive power can be calculated. The Heyland circle is a good approximation for machines with a double cage rotor in normal load conditions.

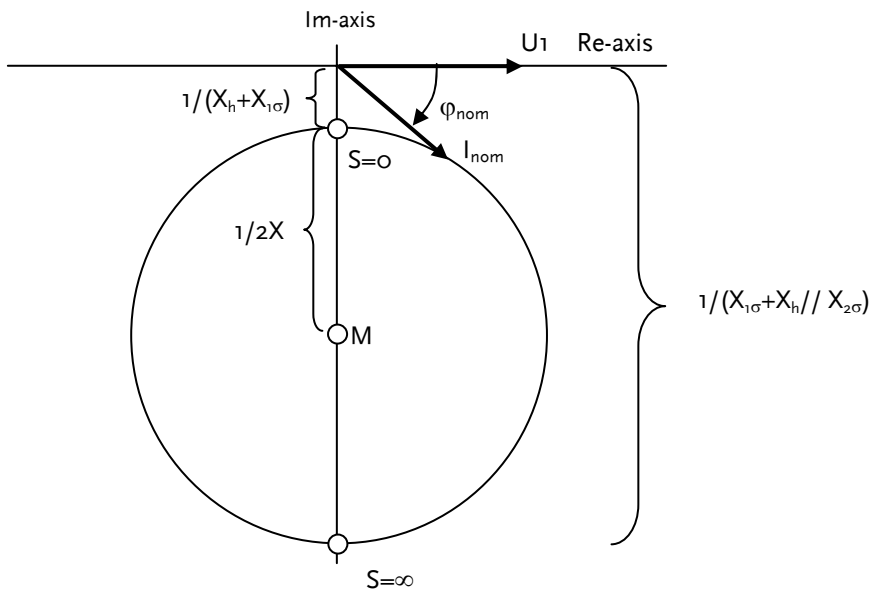


Figure 4 Heyland circle for the model in figure 3

Writing for the admittance:

$$Y = G + jB$$

the Heyland circle equation becomes in Cartesian coordinates:

$$G^2 + \left(B + \frac{1}{X_{1\sigma} + X_h} + \frac{1}{2X} \right)^2 - \left(\frac{1}{2X} \right)^2 = 0 \tag{5}$$

and in polar coordinates:

$$|Y|^2 + 2 \left(\frac{1}{X_{1\sigma} + X_h} + \frac{1}{2X} \right) |Y| \sin(\theta) + \left(\frac{1}{X_{1\sigma} + X_h} + \frac{1}{2X} \right)^2 - \left(\frac{1}{2X} \right)^2 = 0$$

or (6)

$$|Y|^2 + C|Y| \sin(\theta) + D = 0$$

where:

$$G = Y \cos(\theta)$$

$$B = Y \sin(\theta)$$

$$\theta = -\varphi \text{ (because the Phase angle is negative)}$$

Using these equations the model parameters can be deduced from the efficiency and power factor measurements, which are curves as function of the motor load. Next equation shows the admittance as function of power, voltage and power factor, in p.u.:

$$y = \frac{p}{u^2 \cos(\varphi)} \tag{7}$$

with:

$$p = P/S_b$$

$$u = U/U_b$$

Equation (6), written in p.u. yields:

$$y^2 + c \cdot y \sin(\theta) + d = 0 \tag{8}$$

In this equation y^2 follows from (7) and equals: $y \sin(\theta) = - (p/u^2) \tan(\varphi)$.

3.2 The curve fitting process

Equation (8) describes the Heyland circle, from where the asynchronous machine parameters can be deduced. For a normal machine, the model parameters and the Heyland circle are not known. These parameters must be calculated from the efficiency and power factor measurements, as a function of the load power. Using a curve fitting procedure the Heyland circle will be constructed from these measurements. The model parameters follow.

Writing:

$$\begin{aligned} f &= -y^2 \\ g &= y \sin(\theta) \end{aligned}$$

equation (8) changes into a straight line equation in the f,g -system:

$$-f + c \cdot g + d = 0 \tag{9}$$

Since equation (9) represents a straight line, the smallest quadrates fitting will be used to estimate the proper values for c and d from the measurements. The n measurements at different load powers are used. The fitting equations are:

$$\begin{aligned} a_1 &= n \sum g^2 - (\sum g)^2 \\ a_2 &= n \sum f^2 - (\sum f)^2 \\ a_3 &= n \sum fg - \sum f \sum g \\ a_4 &= \sum g^2 \sum f - \sum g \sum fg \\ a_5 &= \sum f^2 \sum g - \sum f \sum fg \end{aligned} \tag{10}$$

Fitting for an optimum of f yields:

$$\begin{aligned} c_1 &= \frac{a_3}{a_1} \\ d_1 &= \frac{a_4}{a_1} \end{aligned} \tag{11}$$

Fitting for an optimum of g yields:

$$\begin{aligned} c_2 &= \frac{a_2}{a_3} \\ d_2 &= -\frac{a_5}{a_3} \end{aligned} \tag{12}$$

The quality of the fitting process is:

$$r^2 = \frac{a_3^2}{a_1 a_2} \tag{13}$$

For a good fit the quality factor r^2 must be between 0.98 and 1.0. The best solution will be given by either c_1 and d_2 or by c_2 and d_2 . Choose the fit that yields the best power factor match at P_{rated} . The curve fitting process needs at least 3 measurement points. One of the measured points must be the at rated power. The no-load point should not be a measured point, since this is an inherent point of the circle. Next diagrams represent the Heyland circles for a good (left) and a bad (right) fit. Ranges 1 and 2 are results of the parameters c_1 and d_1 respectively c_2 and d_2 .

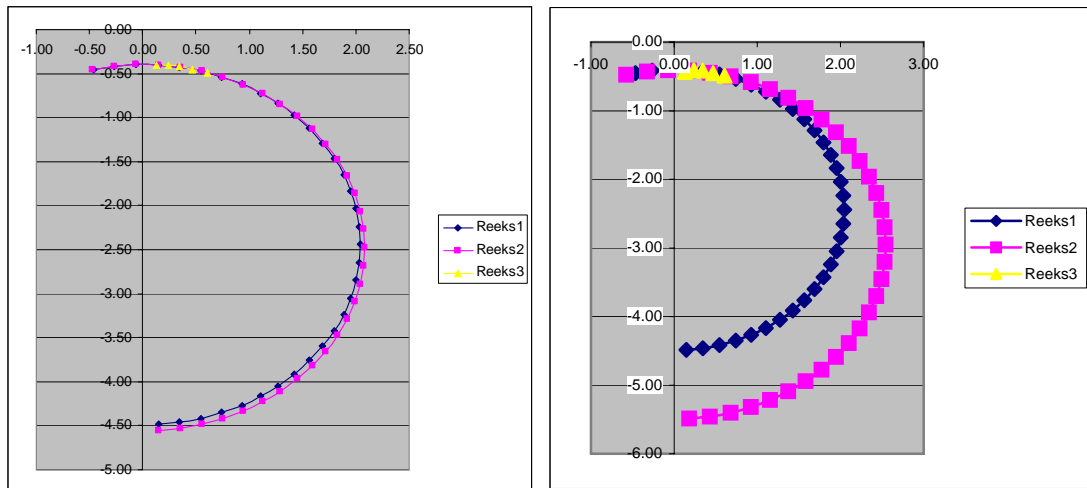


Figure 5 Example of a good (left) and a bad (right) curve fitting

In the diagram Range 3 is the measurement. The diagram shows that the measurements describe a very small part of the circle. This example illustrates that the measurements must be of good accuracy, because the model parameters strongly depend on a good fit. In the example the largest deviations are at low speed. The model only uses the rated speed part of the circle, where the measurements have been made. For a better model estimate the curve fitting should be accepted when the quality of the curve fitting process is between 0.98 and 1.0.

3.3 Standard curves

A number of standard curves have been programmed in Vision Network Analysis. In most cases these curves give acceptable machine models. The efficiency curves are from 60% to 97%, see appendix A. The power factor curves are from 0.6 to 0.92, see appendix B.

These curves assist the modeling of an asynchronous machine. However, it has been recommended to input the exact efficiency and power factor measurements from the manufacturer.

3.4 Determining the active and reactive powers

In the load flow calculation process the asynchronous machine has been modeled with a constant active power, a constant reactive power component and a constant reactive admittance component.

The active power results from the mechanical load and does not depend on the terminal voltage. The reactive power, however, depends on the terminal voltage. The machine model divides the reactive power between a constant reactive power and a constant admittance. In normal load conditions this model approaches a constant current behaviour (reactive).

Next equation describes the machine model for use in the load flow calculations. Distinction must be made between the efficiency of a motor and the efficiency of a generator.

$$\begin{aligned}
 \text{motor} : p_{electric} &= P_{mech,actual} / \eta_{motor} \\
 \text{generator} : p_{electric} &= P_{mech,actual} * \eta_{generator} \\
 q_{machine} &= q_{const} + q_{var} u^2
 \end{aligned} \tag{14}$$

The active electrical power is determined from the active mechanical power.

The reactive power is the sum of a constant part and a variable part (from the constant admittance):

$$p = p_{const} + p_{var} \tag{15}$$

Using a factor α and the load power p_0 at rated voltage, we can write:

$$\begin{aligned}
 p_{const} &= \alpha \cdot p_0 \\
 p_{var} &= (1 - \alpha) \cdot p_0 \cdot \left(\frac{U_b}{U_{nom}} \right)^2 u^2
 \end{aligned} \tag{15a}$$

Resulting in the conductance:

$$g_d = (1 - \alpha) \cdot p_0 \cdot \left(\frac{U_b}{U_{nom}} \right)^2 \tag{16}$$

And resulting in the actual electrical power:

$$p = p_{const} + g_d u^2 \tag{17}$$

In the model the active power is kept constant (as calculated from the actual mechanical power) and as a result the factor α equals 1. The conductance in equation (17) is zero.

Using equation (7) into equation (8) yields a square-root equation, of which only the negative root is of importance (rated machine conditions):

$$q = \frac{u^2}{2} \left(c - \sqrt{c^2 - 4 \left(d + \left(\frac{p}{u^2} \right)^2 \right)} \right) \tag{18}$$

Now the rated power factor $\cos(\varphi)$ can be calculated. The rated reactive power is is:

$$q_{nom} = \frac{1}{2} \left(\frac{U_{nom}}{U_b} \right)^2 \cdot \left(c - \sqrt{c^2 - 4 \left(d + \left(\frac{P_{nom} / S_b}{(U_{nom} / U_b)^2} \right)^2 \right)} \right) \tag{19}$$

Resulting in the rated power factor $\cos(\varphi)$:

$$\cos(\varphi_{nom}) = \frac{1}{\sqrt{1 + \left(\frac{q_{nom}}{p_{nom}}\right)^2}} \quad (20)$$

Also $X_h + X_{1\sigma}$ can be calculated. If $p=0$ en $u=U_{nom}/U_b$, equation (19) changes into:

$$\frac{1}{X_h + X_{1\sigma}} = \frac{q_{p=0, u=nom}}{(U_{nom}/U_b)^2} = \frac{1}{2} \left(c - \sqrt{c^2 - 4d} \right) \quad (21)$$

The variable part of the reactive power can be calculated from the differential to the voltage. The differential partial to the voltage angle is zero. Therefore the differential results only in the differential partial to the voltage magnitude. Combination of equations (17) and (18) yields:

$$q = \frac{1}{2} \left(cu^2 - \sqrt{(c^2 - 4d)u^4 - 4(p_{const} + g_d u^2)^2} \right) \quad (22)$$

If g_d approaches zero, equation (22) changes into:

$$q = \frac{1}{2} \left(cu^2 - \sqrt{(c^2 - 4d)u^4 - 4p_{const}^2} \right) \quad (22a)$$

Differentiation of equation (22a) yields:

$$q' = \frac{\partial q}{\partial u} = 2u \frac{2 \cdot d \cdot u^2 - c \cdot q}{c \cdot u^2 - 2q} \quad (23)$$

$$q'' = \frac{\partial^2 q}{\partial u^2} = 2 \frac{6 \cdot d \cdot u^2 - c(q + 2q'u) + q'^2}{c \cdot u^2 - 2q}$$

In the rated machine conditions the voltage approaches its rated value and equations (22a) and (23) may be written as:

$$q_{u=1} = \frac{1}{2} \left(c - \sqrt{c^2 - 4d - 4p_{const}^2} \right) \quad (24)$$

$$q'_{u=1} = 2 \frac{2 \cdot d - c \cdot q_{u=1}}{c - 2q_{u=1}} \quad (25)$$

$$q''_{u=1} = 2 \frac{6 \cdot d - c(q_{u=1} + 2q'_{u=1}) + q'^2_{u=1}}{c - 2q_{u=1}} \quad (26)$$

Writing this function with a Taylor-series for the point $u=1$ yields:

$$q(u) = q(1) + \frac{u-1}{1!} q'(1) + \frac{(u-1)^2}{2!} q''(1) + \dots \quad (27)$$

$$= q(1) - q'(1) + \frac{1}{2} q''(1) + \dots + u[q'(1) - q''(1) + \dots] + u^2 \left[\frac{1}{2} q''(1) + \dots \right] + \dots$$

Using the series until the second powers, the coefficients of u can be subdivided into those of u^0 and u^2 . The equation then changes into:

$$q(u) \approx q(1) - \frac{1}{2}q'(1) + u^2 \left[\frac{1}{2}q'(1) \right] = q_{const} + u^2 q_{var} \quad (28)$$

Now the division between the constant and variable part of the reactive power can be established. From equation (28) follows, using equation (25), the variable part of the reactive power at rated voltage:

$$q_{var} = \frac{2d - c \cdot q}{c - 2q} \quad (29)$$

As a result the constant part is the difference of the total and the variable parts:

$$q_{const} = q(1) - q_{var} \quad (30)$$

Summarizing, the asynchronous machine has been modeled as a standard constant power load as function of the mechanical power and a reactive power load as function of the terminal voltage. The reactive power variable part has been modeled using a constant admittance.

$$\begin{aligned} p &= p_{const} = f_p(p_{mech}) \\ q &= q_{const} + u^2 q_{var} = f_q(p_{mech}, u) \end{aligned} \quad (31)$$

4 CONCLUSION

The asynchronous machine has been described for study of the behaviour in load flow calculations. Many computer programs model the asynchronous machine as a fixed load. For the implementation in *Vision Network Analysis* a model has been chosen with a better approach of the dependence of mechanic load and actual voltage. The model is used for normal load conditions.

APPENDIX A STANDARD EFFICIENCY CURVES

2 - 4 polen					6 - 12 polen				
P=125%	P=100%	P=75%	P=50%	P=25%	P=125%	P=100%	P=75%	P=50%	P=25%
57	60	61	57	38	58	60	59	55	38
58	61	62	58	40	59	61	60	56	40
59	62	63	59	42	60	62	61	57	42
60	63	64	60	43	61	63	62	58	43
61	64	65	61	45	62	64	63	59	45
62	65	66	62	46	63	65	64	60	46
63	66	67	63	48	64	66	65	61	48
64	67	68	64	49	65	67	66	62	49
65	68	69	65	51	66	68	67	63	51
66	69	70	66	52	67	69	68	64	52
67	70	71	67	54	68	70	69	65	56
68	71	72	68	56	69	71	70	66	56
69	72	73	69	57	70	72	71	67	58
70	73	74	70	59	72	73	73	69	60
71	74	75	71	60	73	74	74	70	62
72	75	76	72	61	74	75	75	71	62
73	76	77	74	63	75	76	76	72	64
74	77	78	75	65	76	77	77	73	64
75	78	79	76	67	77	78	78	74	66
76	79	80	78	69	78	79	79	75	67
77	80	81	79	71	79	80	80	76	68
79	81	82	81	73	80	81	81	77	70
80	82	83	82	74	81	82	82	78	72
81	83	84	83	76	83	83	83	80	74
82	84	85	84	78	84	84	84	81	75
83	85	86	85	79	85	85	85	82	76
86	86	86	85	80	86	86	86	83	78
86	87	87	86	82	87	87	87	84	80
87	88	88	87	83	88	88	88	86	82
88	89	89	88	84	89	89	89	87	83
89	90	90	89	85	90	90	90	88	84
91	91	91	90	86	91	91	91	89	85
92	92	92	91	87	92	92	92	90	86
93	93	93	92	88	93	93	93	91	88
94	94	94	93	89	94	94	94	92	89
95	95	95	94	90	95	95	95	93	90
96	96	96	95	91	96	96	96	94	91
97	97	97	96	92	97	97	97	95	92

APPENDIX B STANDARD COS(ϕ) POWER FACTOR CURVES

P=125%	P=100%	P=75%	P=50%	P=25%
0.64	0.60	0.52	0.40	0.21
0.65	0.61	0.53	0.41	0.22
0.66	0.62	0.54	0.42	0.23
0.68	0.63	0.55	0.43	0.24
0.69	0.64	0.56	0.44	0.25
0.7	0.65	0.57	0.45	0.26
0.71	0.66	0.58	0.46	0.27
0.71	0.67	0.59	0.47	0.27
0.72	0.68	0.60	0.48	0.28
0.72	0.69	0.61	0.49	0.29
0.73	0.70	0.63	0.50	0.30
0.76	0.71	0.64	0.50	0.31
0.78	0.72	0.64	0.50	0.31
0.78	0.73	0.65	0.51	0.31
0.79	0.74	0.66	0.52	0.32
0.79	0.75	0.68	0.56	0.34
0.81	0.76	0.68	0.56	0.35
0.81	0.77	0.69	0.56	0.36
0.82	0.78	0.71	0.58	0.37
0.82	0.79	0.72	0.59	0.38
0.83	0.80	0.73	0.60	0.40
0.84	0.81	0.74	0.63	0.43
0.84	0.82	0.76	0.66	0.46
0.84	0.83	0.78	0.70	0.50
0.85	0.84	0.80	0.71	0.52
0.86	0.85	0.81	0.72	0.54
0.87	0.86	0.82	0.73	0.56
0.88	0.87	0.84	0.76	0.58
0.88	0.88	0.86	0.80	0.60
0.89	0.89	0.87	0.81	0.62
0.9	0.90	0.88	0.82	0.64
0.91	0.91	0.89	0.83	0.66
0.92	0.92	0.90	0.84	0.68